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Cosmic Acceleration and M Theory Cosmology

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This is a short overview of spatially flat (or open) four-dimensional accelerating cosmologies for some simple exponential potentials obtained by string or M theory compactification on some non-trivial curved spaces, which may lead to some striking results, e.g., the observed cosmic acceleration and the scale of the dark energy from first principles.

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1. Introduction

The universe appears to be accelerating, but the reason why is yet to be explored. Cosmologists are trying to confirm the hypothesis of a dark universe, providing two alternative candidates for the dark energy – a positive cosmological constant and a variable Λ -term, like a slowly varying scalar potential; both exert a gravitationally self-repulsive force. One would hold out less hope of understanding the dark energy (or cosmic acceleration) unless or until there is a unified theory with a compelling particle-physics motivation that takes it closer to the bedrock of space and time.

The usual idea in superstring or M theory is that spacetime is a four dimensional nearly flat metric times a small six or seven dimensional internal manifold. The current interest in cosmology with the extra (spatial) dimensions is two fold :

- (i) can inflation and/or observed cosmic acceleration naturally arise from string or M theory compactification on some non-trivial curved internal spaces of time-varying volume ^{1,2,3,4,5,6,7}, explaining the scale of the dark energy ^{8,9}.
- (ii) can one derive a scalar potential from string theory compactification, with static and warped extra dimensions, that has at least one stationary point with $V > 0$ ¹⁰.

One of the obstacles for a warped de Sitter type compactification is a no-go theorem ¹¹. The strong energy condition holds for all $D = 10$ or 11 supergravities. If one takes the extra dimensions to be warped (and static), then for the compactified theory (with or without form-fields in the extra dimensions), one finds $R_{00}^{(4)} \geq 0$, which does not allow the universe to accelerate. It seems less likely that a (false) de

Sitter state one may obtain in warped string theory background, with static extra dimensions, is responsible for cosmic acceleration, because such a vacuum is too unstable for a significant period of inflation to occur. In certain cases, it may be possible to have late time accelerated expansion from a tachyonic potential ¹².

The models like $V \sim \lambda\varphi^4$, $\sim m^2\varphi^2$, or their hybrids, do not deal with the basic puzzles such as that of the initial singularity, nor with the mystery behind dark energy indicated by recent observation. So it might be worthwhile to carefully consider the string or M theory compactification in time dependent backgrounds. I shall argue that the dark energy of the universe is possibly a gravitational scalar potential arising from slowly varying size of extra dimensions, and so it is dynamical.

2. Compactification with fluxes

Let us recall that the four-dimensional part of a $(4 + m)$ dimensional metric space-time is the usual FLRW universe in the standard coordinate (see e.g., ⁵)

$$ds_4^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right), \quad (1)$$

where $k = 0, \pm 1$, $d\Omega_2^2$ the metric on a unit two-sphere, and $a(t)$ the time-dependent scale factor. This gives $R_{00}^{(4)} = -3\ddot{a}(t)/a(t)$ and inflating spacetime implies that $\ddot{a}/a > 0$. The strong energy condition must be violated during inflation. To this end, we may allow the internal space to be time-dependent. As the simplest possibility, one can consider that the M theory spacetime is $\mathcal{M}_4 \times \Sigma_{k_1, m}$:

$$ds_{4+m}^2 = e^{-m\phi(t)} g_{\mu\nu}(x)dx^\mu dx^\nu + r_c^2 e^{2\phi(t)} d\Sigma_{m, k_1}^2. \quad (2)$$

with $\tilde{R}_{ab}(\Sigma_{k_1, m}) = k_1(m-1)\tilde{g}_{ab}/r_c^2$ and $k_1 = -1, 0, +1$. For $k_1 = -1$, $\Sigma = \mathcal{H}^m/\Gamma$ is the compact hyperbolic manifold (CHM). There is a mass gap and the Kaluza-Klein excitations of the metric are heavy; $m_{KK} \gtrsim r_c^{-1} \equiv M \simeq 10^{-3}M_P$ around $\phi \simeq 0$. The field ϕ associated with the size and shape of extra dimensions has an effective potential due to the curvature of the internal space. With the 4-form field $F_{[4]} = 2b\text{vol}(\Sigma_{k_1, m})$, b being the flux parameter, upon the dimensional reduction we get ^{4,6,13}

$$I = M_P^2 \int d^4x \left(\frac{\mathcal{R}[g]}{2} - \frac{m(m+2)}{4} (\partial\phi)^2 + \frac{m(m-1)k_1}{2r_c^2} e^{-(m+2)\phi} - b^2 e^{-3m\phi} \right), \quad (3)$$

where $M_P = 1/\sqrt{8\pi G} \equiv \kappa^{-1}$ is the reduced Planck mass. The size and shape of the extra dimensions are four-dimensional scalars. In the effective 4D theory, with a canonically normalized scalar φ , one considers the Lagrangian density

$$\mathcal{L} = \sqrt{-g} \left(\frac{M_P^2}{2} \mathcal{R}[g] - (\partial\varphi)^2 - 2V(\varphi) \right). \quad (4)$$

The scalar potential $V(\varphi)$, for $k_1 = -1$, takes the following form

$$V(\varphi) = M_P^2 M^2 e^{-2\lambda\kappa\varphi} + \frac{1}{2} M_P^2 f^2 e^{-(6/\lambda)\kappa\varphi}, \quad (5)$$

where $f^2 \equiv b^2 (4/m(m-1))^{3/c^2}$ and $\lambda = \sqrt{\frac{m+2}{m}}$. $V(\varphi)$ could vary gradually as the universe expands, which must be large during inflation, where $\varphi = \varphi_0 \simeq 0$ and $V \sim M_P^2 M^2$. It must also be cancelled to extreme accuracy after inflation to allow the usual radiation and matter dominated eras, where $\varphi \gg \varphi_0$ and $V \sim 10^{-120} M_P^4$.

The equations of motion we need to solve are

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0, \quad (6)$$

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3} (\dot{\varphi}^2 + 2V(\varphi)). \quad (7)$$

The curvature of the spatial part of the metric k/a^2 is related to the total energy density of the universe, so this term cannot be zero precisely. In any case, for $k = 0$, it is convenient to define a new logarithmic time variable τ by

$$d\tau = e^{-\lambda\varphi} dt, \quad \alpha(\tau) = \ln(a(t)) \Rightarrow a(t) = e^{\alpha(\tau)}.$$

With $b = 0$, a class of expanding solutions (in units $M_P = 1$) is ⁹

$$\begin{aligned} \sqrt{3}\alpha &= A_- \ln \cosh M\gamma\tau + A_+ \ln \sinh M\gamma\tau + c_1, \\ \varphi &= A_- \ln \cosh M\gamma\tau - A_+ \ln \sinh M\gamma\tau + c_2, \end{aligned} \quad (8)$$

up to a shift of τ around $\tau = 0$, where $A_{\pm} = (\sqrt{3} \pm \lambda)^{-1}$, $\gamma = \sqrt{(3 - \lambda^2)}/2$. The solution found in ¹⁴ corresponds to $M = 1$. One has $H = \frac{da/dt}{a} = e^{-\lambda\varphi} \alpha' > 0$. The critical value $\lambda = 1$ separates qualitatively the different cosmologies; the solutions with $\lambda > 1$ are only transiently accelerating (even if $b > 0$), while, for $\lambda \leq 1$, the accelerated expansion may be eternal, although $\lambda < 1$ is not obtainable from the compactification of classical supergravities; the M-theory case is $\lambda = 3/\sqrt{7} = 1.13$.

For $k = -1$, to lowest order, the zero-flux solution, with $t \gg M^{-1}$, is

$$a(t) = \frac{\lambda t}{\sqrt{\lambda^2 - 1}} + \frac{\delta_1}{t^\alpha} = a_0 + a_1, \quad \varphi(t) = \frac{1}{\lambda} \ln(M\lambda t) + \left(\frac{a_1}{a_0}\right) \frac{3(1 - \alpha)\lambda}{4}, \quad (9)$$

where $\alpha^2 = (2/\lambda)^2 - 3$. This corresponds to $-1 < w < -1/3$ ($w \equiv p/\rho$). These are not inflationary solutions, in the usual sense, specifically, if one is expecting 60-e-folds or more, rather they may be the solutions responsible for the current cosmic acceleration. It is healthy to keep in mind that, for the late time cosmology (i.e., $t \gg M^{-1}$), there is no need to solve the flatness (i.e., why $\Omega_\varphi \approx 1$) and horizon problems. Rather, one should really look for such a solution around $t \sim M^{-1}$, where $\varphi \simeq \varphi_0 \simeq 0$, with the scale factor $a(t) = H_0^{-1} \sinh H_0 t$, by satisfying $\Delta t \gtrsim 60 H_0^{-1}$.

For a non-zero field strength parameter b (and/or a bulk cosmological constant), which might serve as a source term, the late time (i.e., $t \gg M^{-1}$) cosmic acceleration can be eternal for the coupling $\lambda < \sqrt{2}$, if $k = -1$. This observation is not totally new, which was made before, but rather implicitly, in the work of Halliwell ¹⁵, using a phase-plane method, but the effects of flux were not considered there. For $k = -1$, it is possible that there arise two periods of cosmic acceleration. More discussions with two or more scalar fields will appear in a separate publication.

3. Potential energy = cosmological constant?

Is it possible to tune the M theory motivated potential to the present value of the vacuum energy, so called cosmological constant? This is indeed an interesting question. In the M theory case $m = 7$, and so $c = 3/\sqrt{7}$, one has

$$V = M^2 M_P^2 e^{-\frac{6}{\sqrt{7}} \frac{\varphi - \varphi_0}{M_P}} + \frac{M_P^2 f^2}{2} e^{-2\sqrt{7} \frac{\varphi - \varphi_0}{M_P}} \quad (10)$$

The first exponent $6/\sqrt{7} \approx 2.27 \equiv \lambda_*$ is within the limit where astronomical data might be relevant, $\lambda_* < \sqrt{6} \approx 2.45$. If $M \sim 10^{-3} M_P \sim 10^{15}$ GeV, in order to get $V(\varphi) \simeq 10^{-120} M_P^4$, one requires $\varphi - \varphi_0 \simeq 115$, in 4D Planck units. Such a (large) shift in the value of φ is not impossible during the 13.7 billion years evolution of our universe, but much of this shift should have been during inflation and/or radiation dominated era. As the cosmology around $t = 0$ is not smoothly extrapolated to the present epoch ($t \sim 10^{17}$ sec), the shift in φ with t need not be uniform. The universe appeared to have gone through various phase transitions, from the early pure vacuum dominated era ($\rho_{total} \equiv \rho_\varphi$) to the current era, where $\rho_\varphi \simeq (2/3)\rho_{total}$.

For our purpose, it is rather important to know what is the factor by which the size of the compact internal space may have grown up. In terms of 4D metric, strictly speaking, to a 4D observer, the growth factor (say f_*) is

$$f_* = \left(\frac{m(m-1)}{4} \right)^{1/(m+2)} \exp \left(\frac{2}{\sqrt{m(m+2)}} \frac{\varphi - \varphi_0}{M_P} \right). \quad (11)$$

For $m = 7$ and $\varphi \simeq \varphi_0 + 115$, $f_* \sim 5 \times 10^{12}$, one has $(f_* r_c)^{-1} \sim m_{KK} \sim 0.2$ TeV, which is still interesting value. Let me elaborate a little on the issue of size of the internal space (and its associated mass gap). The authors in reference ⁸ argued that the $m_{KK} \sim 10^{-60}$ (in 4D Planck units), which is closer to the current Hubble scale $H_0 \sim 10^{-33}$ eV and is phenomenologically unacceptable. This estimation is rough and presumably incorrect. The geometric bound on KK modes arising from the hyperbolic compactification scales with r_c^{-1} (or Ricci curvature), other than the diameter, and not generically with the volume of the manifold. For a tentative value of $M \sim 10^{15}$ GeV, $m_{KK} \ll$ TeV only if one allows $V \approx V_0 \ll 10^{-120} M_P^4$. The lower bound for m_{KK} may be pushed up for the $b > 0$ solution, by making $b r_c$ small. To have a qualitative picture, consider the following solution, which follows from $(4+m)$ dimensional field equations,

$$ds_{4+m}^2 = e^{-m\phi(T)} (-S^6 dT^2 + S^2 dx^2) + r_c^2 e^{2\phi(T)} d\mathcal{H}_{m,k_1=-1}^2, \quad (12)$$

$$\phi(T) = \frac{\ln(KL)}{m-1}, \quad S^2 = K^{m/m-1} L^{(m+2)/3(m-1)}, \quad (13)$$

$$K(T) = \frac{r_c}{(m-1)} \frac{\lambda_0 \beta}{\sinh[\lambda_0 \beta |T|]}, \quad L(T) = 2b \sqrt{\frac{m-1}{2m}} \frac{\cosh 3\lambda_0 T}{\lambda_0 \beta}, \quad (14)$$

up to a shift of $(4+m)$ dimensional coordinate time T around $T = 0$, and $\beta \equiv \sqrt{\frac{3(m+2)}{m}}$. The constant λ_0 has the dimension of inverse time. By suitably choosing

the flux parameter b or the curvature radius r_c (or both), it is possible to suppress the growth in the size of the internal space, so that all eigenmodes of Laplace-Beltrami operator on a compact hyperbolic space have wavelengths less than $f_* \times r_c$.

In summary, for a spatially flat FLRW universe, if dominated by an exponential potential of a scalar field, $V = M^2 M_P^2 e^{-2\lambda\kappa\varphi}$, the observed spell of cosmic acceleration is only a transient phenomenon for $\lambda > 1$, and eternal for $\lambda \leq 1$. For all known classical compactifications of 10 or 11d supergravities on some non-trivial curved internal spaces (with or without fluxes) only $\lambda > 1$ arises in practice. So one is led to explore alternatives for the observed cosmic acceleration. For $\lambda > 1$, the cosmic acceleration can be eternal if the curvature of the spatial section of our universe is negative on large scales, although the observable universe seems to be well described by flat Euclidean geometry. It is plausible that the dark energy of the universe is the gravitational scalar potential that arises naturally from slowly varying size of extra dimensions. The spatial curvature k , which might be negative, was significantly important during the early-time inflationary periods.

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